Math 250 2.4 Infinite Limits

Objectives

1) Find and sketch vertical asymptotes
   a. Rational functions
   b. Trig functions
2) Determine infinite limits
   a. One-sided limits from left or right
   b. Two-sided limits

An **infinite limit** is when x approaches a finite value, but y approaches $+\infty$ or $-\infty$.

[Note: This is not a *limit at infinity*, where x approaches $+\infty$ or $-\infty$. Limits *at* infinity are covered in 2.5]

When the limit is infinite, the answer has three parts:

i. $+\infty$ or $-\infty$ or neither
ii. DNE
iii. Reason limit does not exist is UNBOUNDED behavior.

Important note: For a limit to be $+\infty$, *both* one-sided limits must approach $+\infty$. Ditto if $-\infty$.

Properties of Infinite Limits

Let $\lim\limits_{x\to a} f(x)=\infty$ (an infinite limit, y coordinate) and $\lim\limits_{x\to a} g(x)=L$ (a finite limit, y coordinate), where a is a finite number. Then:

a) $\lim\limits_{x\to a}\left[f(x)\pm g(x)\right]=\infty$ because infinity plus or minus any finite number is infinity
b) $\lim\limits_{x\to a}\left[f(x)\cdot g(x)\right]=\infty$ because infinity times any finite number is infinity
c) $\lim\limits_{x\to a}\left[\dfrac{f(x)}{g(x)}\right]=\infty$ because infinity divided by any finite number is infinity
d) $\lim\limits_{x\to a}\left[\dfrac{g(x)}{f(x)}\right]=0$ because a finite number divided by an ever-larger (infinite) number goes to 0

Definition of a Vertical Asymptote

If $\lim\limits_{x\to a} f(x)=\pm\infty$, $\lim\limits_{x\to a^+} f(x)=\pm\infty$, or $\lim\limits_{x\to a^-} f(x)=\pm\infty$, the line $x=a$ is called a vertical asymptote.

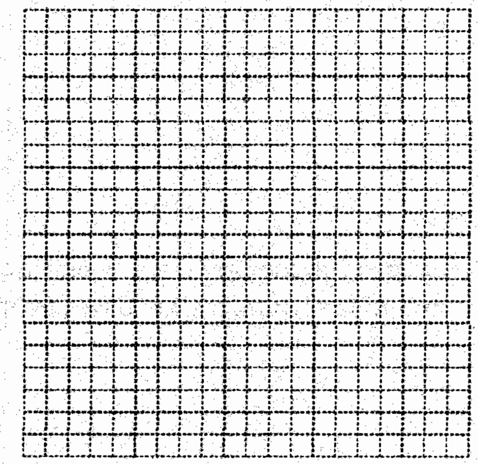
Hint

$\dfrac{6}{0}$ is undefined, but $\dfrac{0}{0}$ is indeterminate. If direct substitution gives an undefined (finite number divided by zero) function value, consider the graph and look for an infinite limit.

Examples and Practice:

Find the equations of any vertical asymptotes, sketch graphs, and use graphs to find the limits

1) $f(x) = \dfrac{-5}{x^3 - 2x^2 - 4x + 8}$



a. $\lim_{x \to -2^+} f(x)$

b. $\lim_{x \to -2^-} f(x)$

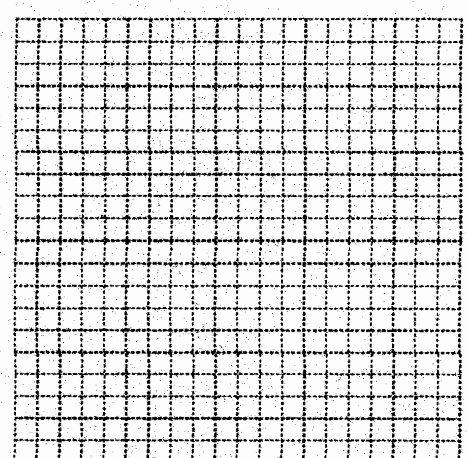
c. $\lim_{x \to -2} f(x)$

d. $\lim_{x \to 2^+} f(x)$

e. $\lim_{x \to 2^-} f(x)$

f. $\lim_{x \to 2} f(x)$

2) $h(x) = \dfrac{-5x + 10}{x^2 - 4}$



a. $\lim_{x \to 2^+} h(x)$

b. $\lim_{x \to 2^-} h(x)$

c. $\lim_{x \to 2} h(x)$

d. $\lim_{x \to -2^+} h(x)$

e. $\lim_{x \to -2^-} h(x)$

f. $\lim_{x \to -2} h(x)$

3) $k(x) = \sec\frac{x}{2}$ on $[-2\pi, 2\pi]$

a. $\lim_{x \to \pi^+} k(x)$

b. $\lim_{x \to \pi^-} k(x)$

c. $\lim_{x \to \pi} k(x)$

4) $m(x) = \dfrac{-(x+2)}{\cot\left(\frac{\pi x}{4}\right)}$ on $[-10, 10]$

a. $\lim_{x \to -6^+} m(x)$

b. $\lim_{x \to -6^-} m(x)$

c. $\lim_{x \to -6} m(x)$

d. BONUS: Approximate $\lim_{x \to -2} m(x)$ to nearest hundredth.

TEST YOURSELF: Review of Limits

5) $f(x) = \begin{cases} \dfrac{1}{x+3} & x < 0 \\ -\dfrac{7}{16}x^2 + 4 & 0 \le x < 4 \\ \sqrt{x} - 5 & x > 4 \end{cases}$

a. $f(-3)$

b. $\lim_{x \to -3} f(x)$

c. $f(0)$

d. $\lim_{x \to 0} f(x)$

e. $f(4)$

f. $\lim_{x \to 4} f(x)$

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① $f(x) = \dfrac{-5}{x^3-2x^2-4x+8}$

Factor denominator: 4 terms, factor by grouping

$\underbrace{x^3-2x^2}_{GCF\ x^2} \underbrace{-4x+8}_{GCF\ -2}$

$= x^2(x-2) - 4(x-2)$

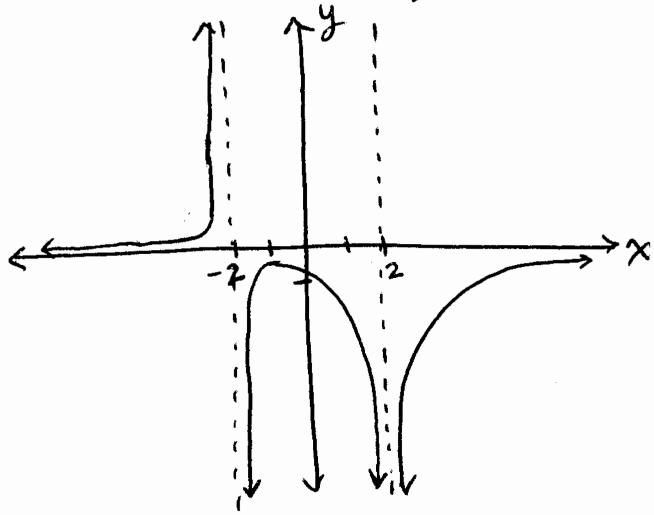
GCF $(x-2)$

$= (x-2)(x^2-4)$ difference of squares

$= (x-2)(x-2)(x+2)$

$\boxed{\text{vertical asymptotes } x=2,\ x=-2}$

Sketch graph. (Using GC $Y_1 = -5/(x^3-2x^2-4x+8)$)



a) $\lim_{x\to -2^+} f(x) = \boxed{\begin{array}{l} -\infty \\ \text{DNE} \\ \text{unbounded} \end{array}}$

limit from right as $x \to 2$

-2

b) $\lim_{x\to -2^-} f(x) = \boxed{\begin{array}{l} +\infty \\ \text{DNE} \\ \text{unbounded} \end{array}}$

limit from left as $x \to -2$

-2

c) $\lim_{x\to -2} f(x) = \boxed{\begin{array}{l} \text{DNE} \\ L \neq R \\ \text{unbounded} \end{array}}$ <-- because $\underbrace{\lim_{x\to -2^+} f(x)}_{-\infty} \neq \underbrace{\lim_{x\to -2^-} f(x)}_{+\infty}$

Remember: List ALL reasons that a limit does not exist.

d) $\lim_{x\to 2^+} f(x) = \boxed{\begin{array}{l} -\infty \\ \text{DNE} \\ \text{unbounded} \end{array}}$

e) $\lim_{x\to 2^-} f(x) = \boxed{\begin{array}{l} -\infty \\ \text{DNE} \\ \text{unbounded} \end{array}}$

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f) $\lim_{x \to 2} f(x) = \boxed{\begin{array}{l} -\infty \\ \text{DNE} \\ \text{unbounded} \end{array}}$ $\leftarrow$ because $\underbrace{\lim_{x \to 2^+} f(x)}_{-\infty} = \underbrace{\lim_{x \to 2^-} f(x)}_{-\infty}$

② $h(x) = \dfrac{-5x+10}{x^2-4}$

Factor completely.

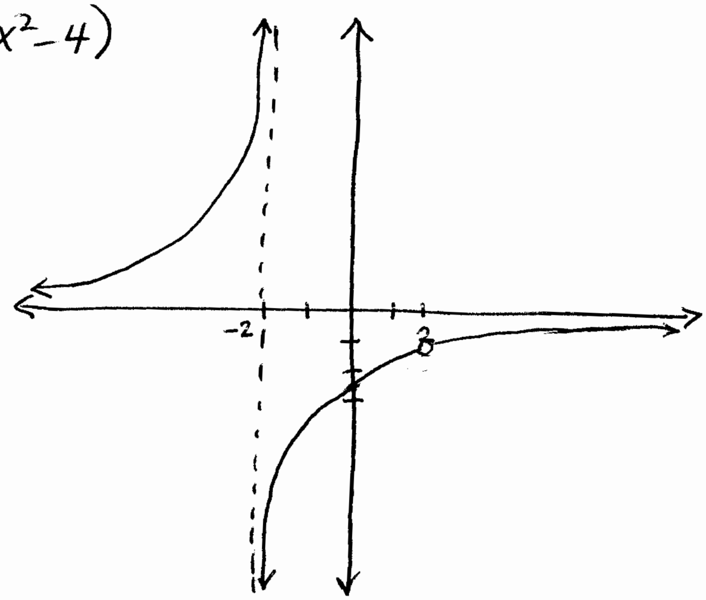
$h(x) = \dfrac{-5(x-2)}{(x-2)(x+2)}$ $\leftarrow$ $x \neq 2$ and $x \neq -2$
Neither value is in domain.

$h(x) = \begin{cases} \dfrac{-5}{(x+2)} & \text{for all } x \neq 2 \\ \text{undefined} & x = 2 \end{cases}$

$\dfrac{(x-2)}{(x-2)}$ can be divided out, so long as we remember that $h(2)$ is undefined.
Because $\dfrac{(x-2)}{(x-2)} = 1$, $x=2$ is a hole.

$\boxed{\text{vertical asymptote } x=-2 \text{ only}}$

Graph. In GC $y_1 = (-5x+10)/(x^2-4)$



a) $\lim_{x \to 2^+} h(x)$ $\dashrightarrow$ $\frac{0}{0}$ ☹

$= \lim_{x \to 2^+} \dfrac{-5\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$

$= \lim_{x \to 2^-} \dfrac{-5}{x+2}$

$= \dfrac{-5}{2+2}$

$= \boxed{\dfrac{-5}{4}}$

b) $\lim_{x \to 2^-} h(x) = \boxed{\dfrac{-5}{4}}$ by same reasoning

c) $\lim_{x \to 2} h(x) = \boxed{\dfrac{-5}{4}}$

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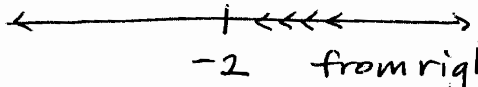
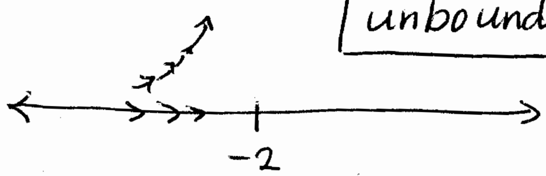
d) $\lim_{x \to -2^+} h(x) = \boxed{\begin{array}{l} -\infty \\ \text{DNE} \\ \text{unbounded} \end{array}}$



from right

graph goes down

e) $\lim_{x \to -2^-} h(x) = \boxed{\begin{array}{l} +\infty \\ \text{DNE} \\ \text{unbounded} \end{array}}$



from left, graph goes up

f) $\lim_{x \to -2} h(x) = \boxed{\begin{array}{l} \text{DNE} \\ \text{unbounded} \\ L \neq R \end{array}}$

③ $k(x) = \sec\left(\frac{x}{2}\right) = \dfrac{1}{\cos\left(\frac{x}{2}\right)}$

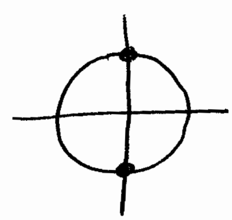
period: $\frac{x}{2} = 2\pi$ argument = original period of $\sec\theta$ or $\cos\theta$

$x = 4\pi$ solve equation for x

asymptotes: $\cos\left(\frac{x}{2}\right) = 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$



$\frac{x}{2} = \frac{\pi}{2}$ set argument = location of zero on untransformed function

$x = \pi$

repeat: $\frac{x}{2} = \frac{3\pi}{2}$

$x = 3\pi$ (out of domain)

$\cos\theta = 0$ also when $\theta = -\frac{\pi}{2}, -\frac{3\pi}{2}$

repeat: $\frac{x}{2} = -\frac{\pi}{2}$

$x = -\pi$

repeat: $\frac{x}{2} = -\frac{3\pi}{2} \Rightarrow x = -3\pi$ (out of domain)

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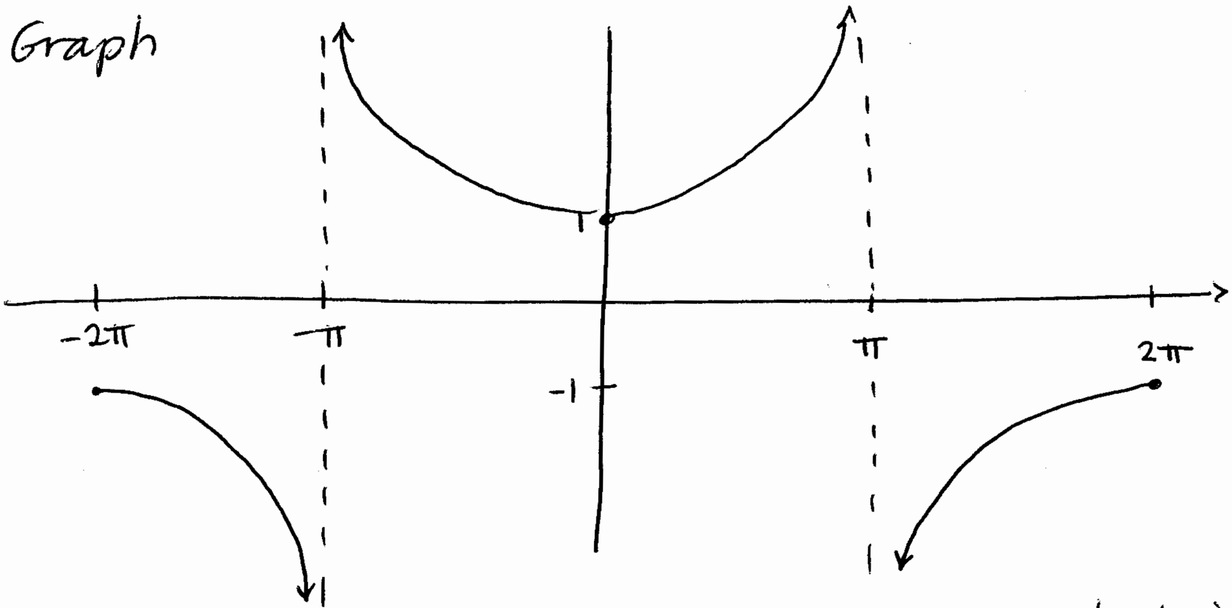
Graph



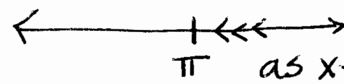
$\sec\left(\frac{0}{2}\right) = \sec 0 = \frac{1}{\cos 0} = \frac{1}{1} = 1$

$\sec\left(\frac{2\pi}{2}\right) = \sec \pi = \frac{1}{\cos \pi} = \frac{1}{-1} = -1$

$\sec\left(\frac{-2\pi}{2}\right) = \sec(-\pi) = \frac{1}{\cos(-\pi)} = \frac{1}{-1} = -1$

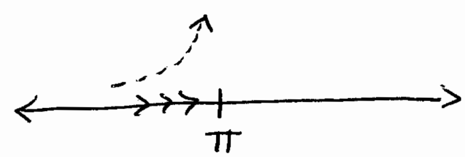
check in GC:
$y_1 = 1/\cos(x/2)$
$\boxed{\text{ZOOM}}$
7.TRIG

a) $\lim_{x \to \pi^+} k(x) = \boxed{\begin{array}{l} -\infty \\ \text{DNE} \\ \text{unbounded} \end{array}}$



as $x \to \pi$ from right graph goes down.

b) $\lim_{x \to \pi^-} k(x) = \boxed{\begin{array}{l} +\infty \\ \text{DNE} \\ \text{unbounded} \end{array}}$



as $x \to \pi$ from left graph goes up.

c) $\lim_{x \to \pi} k(x) = \boxed{\begin{array}{l} \text{DNE} \\ L \neq R \\ \text{unbounded} \end{array}}$

$\lim_{x \to \pi^+} k(x) \neq \lim_{x \to \pi^-} k(x)$

$-\infty \neq +\infty$

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④ $m(x) = \dfrac{-(x+2)}{\cot\left(\frac{\pi x}{4}\right)}$ on $[-10, 10]$

$\cot\theta = \dfrac{1}{\tan\theta}$ means $\dfrac{1}{\cot\theta} = \tan\theta$

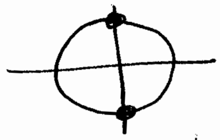
$m(x) = -(x+2) \cdot \tan\left(\dfrac{\pi x}{4}\right)$

period: $\dfrac{\pi x}{4} = \pi$ set argument = original period

$x = \dfrac{4\pi}{\pi} = 4$ solve for x.

period $\cot\left(\frac{\pi x}{4}\right)$ is 4 units. (no π).

asymptotes: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ means $\cos\theta = 0$ gives asymptote.



$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{-\pi}{2}, \frac{-3\pi}{2}$

But $\theta = \dfrac{\pi x}{4}$. Set this expression equal to locations of asymptotes on original.

$\dfrac{\pi x}{4} = \dfrac{\pi}{2}$ $\qquad$ $\dfrac{\pi x}{4} = \dfrac{3\pi}{2}$ $\qquad$ $\dfrac{\pi x}{4} = \dfrac{-\pi}{2}$ $\qquad$ $\dfrac{\pi x}{4} = \dfrac{-3\pi}{2}$

$x = \dfrac{\pi}{2} \cdot \dfrac{4}{\pi}$ $\qquad$ $x = \dfrac{3\pi}{2} \cdot \dfrac{4}{\pi}$ $\qquad$ $x = \dfrac{-\pi}{2} \cdot \dfrac{4}{\pi}$ $\qquad$ $x = \dfrac{-3\pi}{2} \cdot \dfrac{4}{\pi}$

$x = 2$ $\qquad$ $x = 6$ $\qquad$ $x = -2$ $\qquad$ $x = -6$

spacing between asymptotes = period

$\boxed{\text{vertical asymptotes} \quad -10, -6, -2, 2, 6, 10}$

zero when $m(x) = 0$ (x-intercept)

$\dfrac{-(x+2)}{\cot\left(\frac{\pi x}{4}\right)} = 0$

$-(x+2) = 0\left(\cot\dfrac{\pi x}{4}\right) = 0$

$x + 2 = 0$

$x = -2$

WOAH! How can $x = -2$ be both a zero and an asymptote? It can't!

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Additional zeros

$$m(x) = -(x+2)\cdot \underbrace{\tan\left(\frac{\pi x}{4}\right)}_{\text{when this factor}=0}$$

$$\tan\left(\frac{\pi x}{4}\right) = \frac{\sin\left(\frac{\pi x}{4}\right)}{\cos\left(\frac{\pi x}{4}\right)} = 0$$

when $\sin\left(\frac{\pi x}{4}\right) = 0$

$\sin\theta = 0$

$\theta = 0, \pi, -\pi$

$\theta = 2\pi, -2\pi$

$\frac{\pi x}{4} = 0$ $\quad$ $\frac{\pi x}{4} = \pi$ $\quad$ $\frac{\pi x}{4} = -\pi$

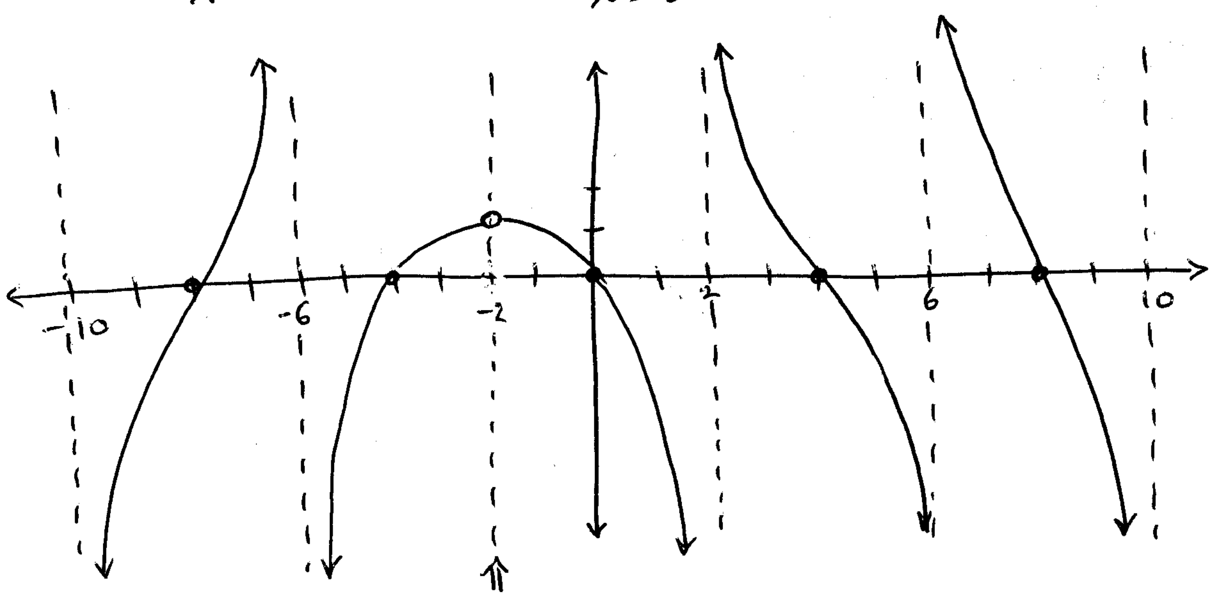
$x = 0$ $\quad$ $x = 4$ $\quad$ $x = -4$

also $\frac{\pi x}{4} = +2\pi$ $\quad$ $\frac{\pi x}{4} = -2\pi$

$x = 8$ $\quad$ $x = 8$

Use GC to fill in details:

$y_1 = -(x+2) * \tan\left(\frac{\pi x}{4}\right)$

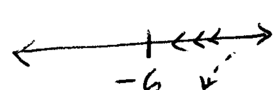


??

Table $x=-2$ ERROR $\longrightarrow$ hole!

But it's NOT $y=0$

a) $\lim_{x\to -6^+} m(x) = \boxed{\begin{array}{l} -\infty \\ \text{DNE} \\ \text{unbounded} \end{array}}$



as $x \to -6$ from right graph goes down

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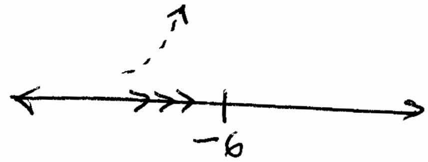
b) $\lim_{x \to -6^-} m(x) = \boxed{\begin{array}{l} +\infty \\ \text{DNE} \\ \text{unbounded} \end{array}}$



as $x \to -6$ from left graph goes up.

c) $\lim_{x \to -6} m(x) = \boxed{\begin{array}{l} \text{DNE} \\ L \neq R \\ \text{unbounded} \end{array}}$

because $\lim_{x \to -6^+} m(x) \neq \lim_{x \to -6^-} m(x)$.

d) $\lim_{x \to -2} m(x)$ approximate using a numerical table

x	-1.9	-1.99	-1.999	-1.9999
$m(x)$	1.2706	1.2732	1.2732	1.2732

x	-2.1	-2.01	-2.001	-2.0001
$m(x)$	1.2706	1.2732	1.2732	1.2732

$\lim_{x \to -2} m(x) \approx \boxed{1.27}$

Notice: If you put $x = -2$ into a table for $y = -(x+2) * \tan\left(\frac{\pi x}{4}\right)$ you get an error, confirming that $x = -2$ is not in the domain of $m(x)$.

To find $\lim_{x \to -2} m(x)$ analytically, you will need L'Hôpital's Rule, which is taught in Math 251.

Math 250

(5) $f(x) = \begin{cases} \frac{1}{x+3} & x<0 \\ -\frac{7}{16}x^2+4 & 0 \le x < 4 \\ \sqrt{x} - 5 & x > 4 \end{cases}$

Graph.

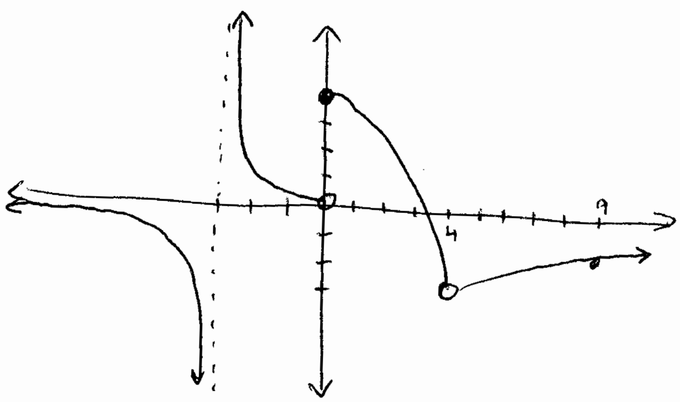
In GC: $y_1 = (1/(x+3))(x<0)$

↑ $\boxed{\text{2nd}}\boxed{\text{MATH}}$ is $\boxed{\text{TEST}}$

$y_2 = (-7/16 * x^2 + 4)((x \ge 0) \text{ and } (x<4))$

↑ $\boxed{\text{2nd}}\boxed{\text{MATH}}\boxed{>}$ is $\boxed{\text{LOGIC}}$

$y_3 = (\sqrt{\ }(x) - 5)(x>4)$



a) $f(-3)$ $\boxed{\text{undefined}}$ $\frac{1}{-3+3} = \frac{1}{0}$

b) $\lim_{x\to -3} f(x) = \boxed{\text{DNE, } L \ne R\text{, unbounded}}$

c) $f(0) = -\frac{7}{16}(0)^2+4 = \boxed{4}$

d) $\lim_{x\to 0} f(x) = \boxed{\text{DNE, } L\ne R}$

e) $f(4) = \boxed{\text{undefined}}$ 4 is not included in any domain.

f) $\lim_{x\to 4} f(x)$ looks like $\sqrt{4} - 5 = 2-5 = -3$

or $-\frac{7}{16}(4)^2 + 4 = -7+4 = -3$ $\boxed{-3}$

Additional Examples

Examples and Practice:

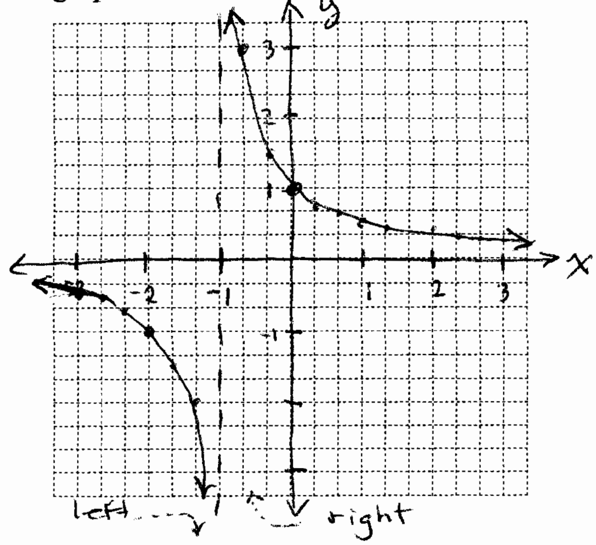
Find the equations of any vertical asymptotes, sketch graphs, and use graphs to find the limits

1) $f(x)=\dfrac{1}{(x+1)}$ $\quad x=-1$ asymptote

a. $\displaystyle\lim_{x\to -1^+}\frac{1}{x+1}=+\infty$, DNE unbounded

b. $\displaystyle\lim_{x\to -1^-}\frac{1}{x+1}=-\infty$, DNE unbounded

c. $\displaystyle\lim_{x\to -1}\frac{1}{x+1}=$ DNE, $L\neq R$, unbounded

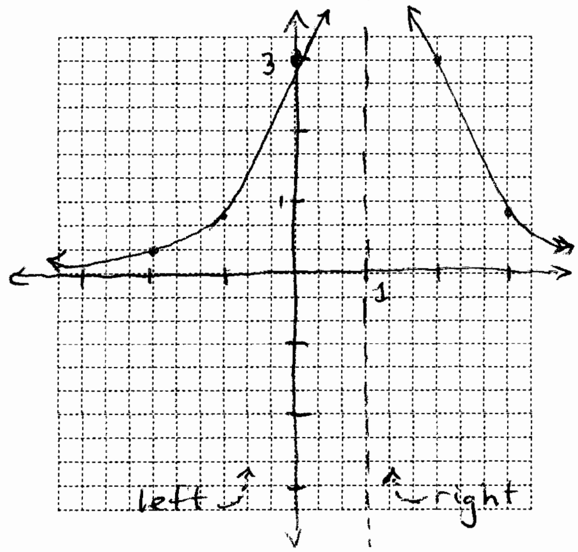


2) $g(x)=\dfrac{3}{(x-1)^2}$ $\quad x=1$ asymptote

a. $\displaystyle\lim_{x\to 1^+}\frac{3}{(x-1)^2}=+\infty$, DNE unbounded

b. $\displaystyle\lim_{x\to 1^-}\frac{3}{(x-1)^2}=+\infty$ DNE unbounded

c. $\displaystyle\lim_{x\to 1}\frac{3}{(x-1)^2}=+\infty$ DNE unbounded



3) $h(x)=\dfrac{-5}{x^2-4}$ $\quad x^2-4=0 \quad x=2,-2$ asymptotes

a. $\displaystyle\lim_{x\to 2^+}\frac{-5}{x^2-4}=-\infty$, DNE, unbounded

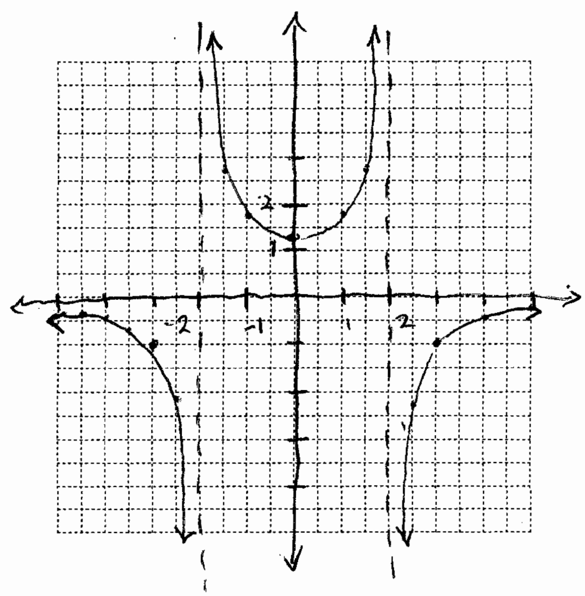
b. $\displaystyle\lim_{x\to 2^-}\frac{-5}{x^2-4}=+\infty$, DNE unbounded

c. $\displaystyle\lim_{x\to 2}\frac{-5}{x^2-4}=$ DNE, $L\neq R$, unbounded

d. $\displaystyle\lim_{x\to -2^+}\frac{-5}{x^2-4}=+\infty$, DNE unbounded

e. $\displaystyle\lim_{x\to -2^-}\frac{-5}{x^2-4}=-\infty$, DNE unbounded

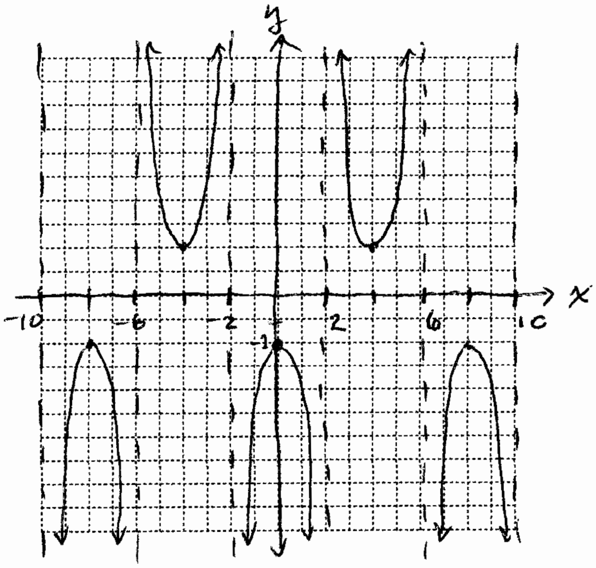
f. $\displaystyle\lim_{x\to -2}\frac{-5}{x^2-4}=$ DNE, $L\neq R$, unbounded



period $\frac{\pi x}{4} = 2\pi \Rightarrow x = 8$

asymptotes $\frac{\pi x}{4} = \frac{\pi}{2}$

4) $k(x) = -2\sec\frac{\pi x}{4}$ $\Rightarrow x = 2 + 4k,\ k \in \mathbb{Z}$

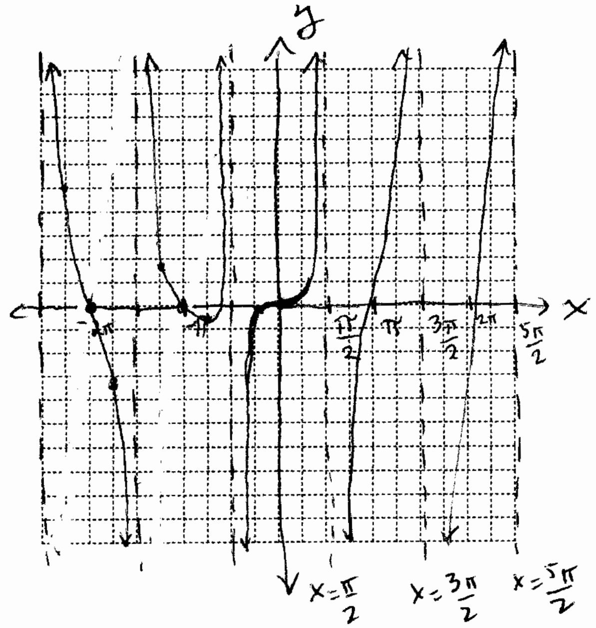


a. $\lim_{x\to 2^+} -2\sec\frac{\pi x}{4} = +\infty$, DNE unbounded

b. $\lim_{x\to 2^-} -2\sec\frac{\pi x}{4} = -\infty$, DNE unbounded

c. $\lim_{x\to 2} -2\sec\frac{\pi x}{4} =$ DNE, L≠R, unbounded

5) $m(x) = \frac{x+2}{\cot x} = (x+2)\tan x$ asymptotes $x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$



a. $\lim_{x\to \frac{\pi}{2}^+} \frac{x+2}{\cot x} = -\infty$, DNE unbounded

b. $\lim_{x\to \frac{\pi}{2}^-} \frac{x+2}{\cot x} = +\infty$ DNE unbounded

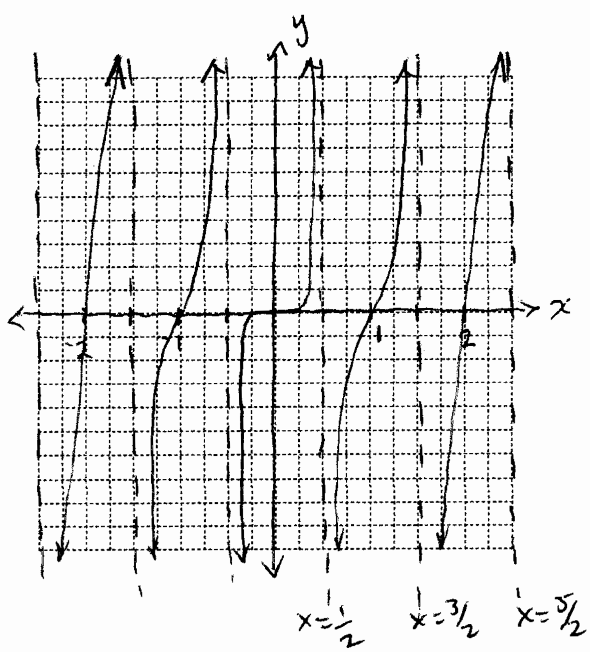
c. $\lim_{x\to \frac{\pi}{2}} \frac{x+2}{\cot x} =$ DNE L≠R unbounded

d. $\lim_{x\to 0} \frac{x+2}{\cot x} = (0+2)\tan 0 = 2(0) = 0$

period $\pi x = \pi$
period 1

asymptotes $\pi x = \frac{\pi}{2}$

6) $n(x) = x^2\tan(\pi x)$ $x = \frac{1}{2} + k \quad k \in \mathbb{Z}$



a. $\lim_{x\to \frac{1}{2}^+} x^2\tan(\pi x) = -\infty$, DNE unbounded

b. $\lim_{x\to \frac{1}{2}^-} x^2\tan(\pi x) = +\infty$, DNE unbounded

c. $\lim_{x\to \frac{1}{2}} x^2\tan(\pi x) =$ DNE L≠R unbounded

Math 250

$\boxed{\text{Summary Examples}}$ – Infinite Limits

Vertical Asymptotes and infinite limits

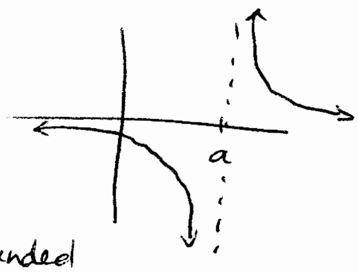
An infinite limit means the y-value approaches infinity, either $+\infty$ or $-\infty$.

Note: $a>0$

(7) a. $\lim\limits_{x\to a} \frac{1}{x-a} = \boxed{\begin{matrix}\text{DNE}\\ L\neq R\end{matrix}}$

b. $\lim\limits_{x\to a^+} \frac{1}{x-a} = \boxed{+\infty}$ DNE unbounded

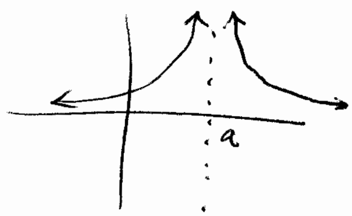
c. $\lim\limits_{x\to a^-} \frac{1}{x-a} = \boxed{-\infty}$ DNE unbounded



(8) a. $\lim\limits_{x\to a} \frac{1}{(x-a)^2} = \boxed{+\infty}$ DNE unbounded

b. $\lim\limits_{x\to a^+} \frac{1}{(x-a)^2} = \boxed{+\infty}$ DNE unbounded

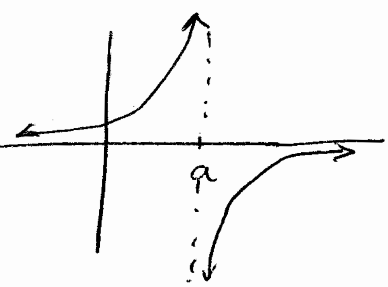
c. $\lim\limits_{x\to a^-} \frac{1}{(x-a)^2} = \boxed{+\infty}$ DNE unbounded



(9) a. $\lim\limits_{x\to a} \frac{-1}{x-a} = \boxed{\text{DNE}}$ $L\neq R$ unbounded

b. $\lim\limits_{x\to a^+} \frac{-1}{x-a} = \boxed{-\infty}$ DNE unbounded

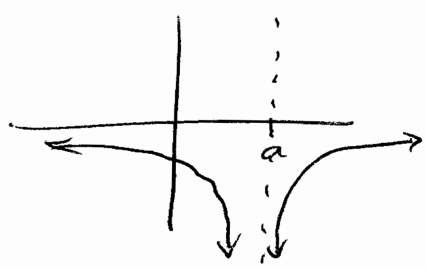
c. $\lim\limits_{x\to a^-} \frac{-1}{x-a} = \boxed{+\infty}$ DNE unbounded



(10) a. $\lim\limits_{x\to a} \frac{-1}{(x-a)^2} = \boxed{-\infty}$ DNE unbounded

b. $\lim\limits_{x\to a^+} \frac{-1}{(x-a)^2} = \boxed{-\infty}$ DNE unbounded

c. $\lim\limits_{x\to a^-} \frac{-1}{(x-a)^2} = \boxed{-\infty}$ DNE unbounded

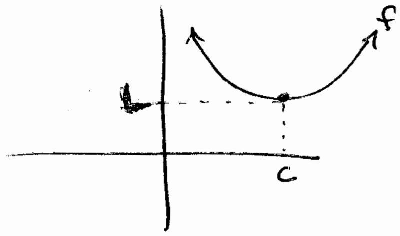


> You may use the graph or numerical evidence to find limits near vertical asymptotes.

Math 250 $\boxed{\text{General Summary}}$ – Many Limits.

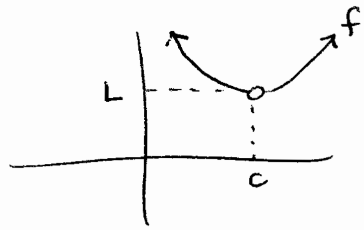
Estimate the limits using graphs.

(11) $\lim\limits_{x \to c} f(x) = \boxed{L}$



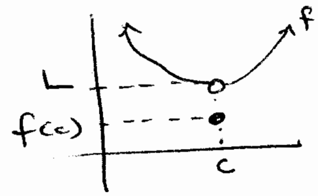
$f(c) = L$

(12) $\lim\limits_{x \to c} f(x) = \boxed{L}$



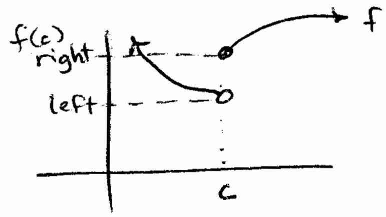
f(c) is not defined!

(13) $\lim\limits_{x \to c} f(x) = \boxed{L}$



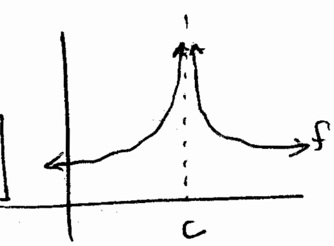
$f(c) \neq L$

(14) $\lim\limits_{x \to c} f(x) = \boxed{\begin{array}{l}\text{DNE}\\ L \neq R\end{array}}$



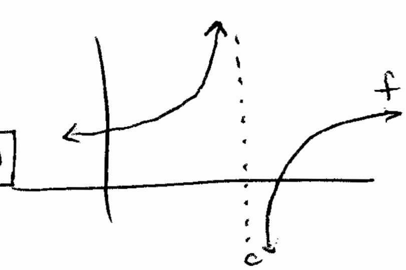
"DNE" is an abbreviation for "DOES NOT EXIST"
If you write this as answer, ALWAYS include the reason.

(15) $\lim\limits_{x \to c} f(x) = \boxed{+\infty}$ $\boxed{\begin{array}{l}\text{DNE}\\ \text{unbounded}\end{array}}$



f(c) not defined

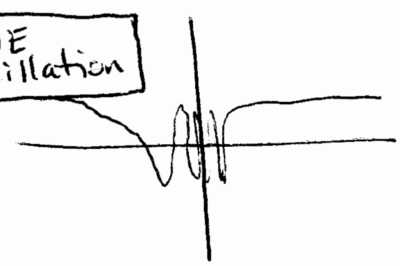
(16) $\lim\limits_{x \to c} f(x) = \boxed{\begin{array}{l}\text{DNE}\\ L \neq R\end{array}}$ $\boxed{\text{unbounded}}$



Note: Some books say that infinite values are not valid limits, and call these DNE. Anton does not do this, but the previous book, Larson, did.

f(c) not defined.

(17) $\lim\limits_{x \to 0} \cos\left(\frac{1}{x}\right) = \boxed{\begin{array}{l}\text{DNE}\\ \text{oscillation}\end{array}}$



There are only 3 reasons a limit DNE

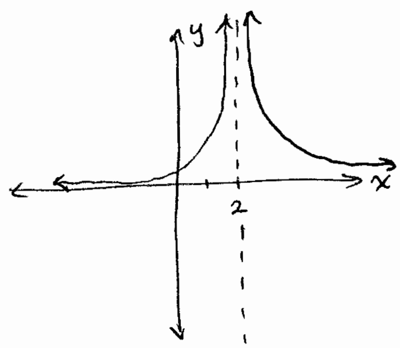
1. Left ≠ Right
2. Oscillation
3. Unbounded (infinite)

Math 250

15

$$\lim_{x \to 2} \frac{1}{|x-2|}$$

a)



$$y = f(x) = \frac{1}{|x-2|}$$

b) $f(2) = \frac{1}{|2-2|} = \frac{1}{|0|} = \frac{1}{0}$ $\boxed{\text{undefined}}$

c) as $x \to 2$ from right, $y \to +\infty$
as $x \to 2$ from left, $y \to +\infty$

$$\lim_{x \to 2} \frac{1}{|x-2|} = \boxed{+\infty, \text{ DNE, unbounded}}$$

We write $+\infty$ since L=R (infinite limit) But any infinite answer grows without bound, so it's unbounded.

Reason #2 Limit DNE: unbounded

d)

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	10	100	1000		1000	100	10

as $x \to 2$ from left $y \to +\infty$

as $x \to 2$ from right $y \to +\infty$.

e) $\lim_{x \to 2} f(x) = \boxed{+\infty, \text{ DNE, unbounded}}$

Math 250

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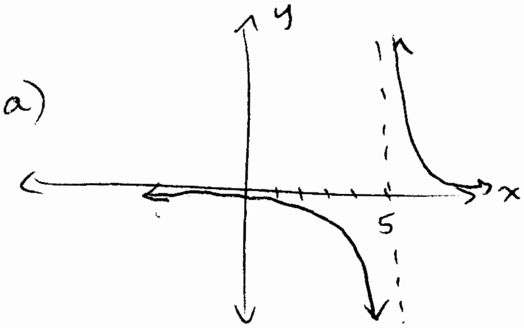
$$\lim_{x \to 5} \frac{2}{x-5}$$

a) 

$y = f(x) = \frac{2}{x-5}$

b) $f(5) = \frac{2}{5-5} = \frac{2}{0}$ $\boxed{\text{undefined}}$

c) $\lim_{x \to 5} f(x) = \boxed{\text{DNE, } L \neq R \text{ and Unbounded}}$

d)

x	4.9	4.99	4.999	5	5.001	5.01	5.1
$f(x)$	-20	-200	-2000		2000	200	20

as $x \to 5$ from left, $y \to -\infty$

as $x \to 5$ from right, $y \to +\infty$

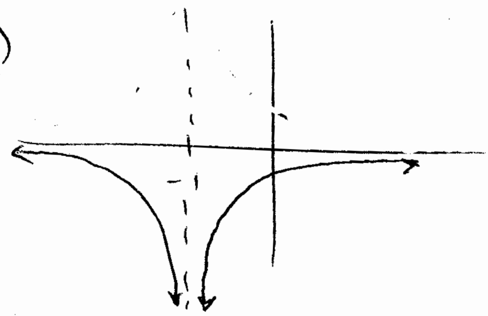
e) $\lim_{x \to 5} f(x) = \boxed{\text{DNE, } L \neq R \text{ and unbounded}}$

Math 250

20

$$\lim_{x \to -1} \frac{-(x+1)}{(x+1)^3}$$

a)



$$y = f(x) = \frac{-(x+1)}{(x+1)^3} = \frac{-1}{(x+1)^2}$$

b) $f(-1) = \frac{-(-1+1)}{(-1+1)^3} = \frac{-0}{0^3} = \boxed{\frac{0}{0} \text{ indeterminate}}$

c) $\lim_{x \to -1} f(x) = \boxed{-\infty, \text{ DNE unbounded}}$

d)

x	-1.1	-1.01	-1.001	-1	$-.999$	$-.99$	$-.9$
$f(x)$	-100	-10000	$-1{,}000{,}000$		$-1{,}000{,}000$	-10000	-100

as $x \to -1$ from left, $y \to -\infty$

as $x \to -1$ from right, $y \to -\infty$

e) $\lim_{x \to -1} f(x) = \boxed{-\infty, \text{ DNE unbounded}}$